INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.MATH - Third Year, 2021-22
Statistics - III, Backpaper Examination, January, 2022
Time: 2 Hours **Total Marks: 50**
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You may freely consult the lecture notes, but no other books or resources may be consulted. You may use any of the results stated and discussed in the lecture notes, by stating them explicitly. Results from the assignments may not be used without establishing them. Calculators may be used.

1. Let $\mathbf{Y} \sim N_n(\mathbf{0}, \sigma^2 I_n)$. Find the conditional distribution of $\mathbf{Y}'\mathbf{Y}$ given $\mathbf{a}'\mathbf{Y} = 0$ where $\mathbf{a}$ is a non-zero constant vector. [8]

2. Consider the model $\mathbf{Y} = \mathbf{X}\beta + \epsilon$, where $\mathbf{X}_{n\times p}$ has $\mathbf{1}$ as its first column and rank $r \le p$, and $\epsilon \sim N_n(\mathbf{0}, \sigma^2 I_n)$.
(a) If $\hat{\beta}$ is the least squares estimator of β, show that $(\hat{\beta} - \beta)'\mathbf{X}'\mathbf{X}(\hat{\beta} - \beta)$ is distributed independently of the residual sum of squares.
(b) Find the maximum likelihood estimator of σ^2. Is it unbiased?
(c) Consider the case when p is 2. When do we have independence of $\hat{\beta}_0$ and $\hat{\beta}_1$? $[6 + 6 + 6]$

3. Consider the following model:
$y_1 = \theta + \gamma + \epsilon_1$
$y_2 = \theta + \phi + \epsilon_2$
$y_3 = 2\theta + \phi + \gamma + \epsilon_3$
$y_4 = \phi - \gamma + \epsilon_4$,
where ϵ_i are uncorrelated having mean 0 and variance σ^2.
(a) Show that $\gamma - \phi$ is estimable. What is its BLUE?
(b) Find the residual sum of squares. What is its degrees of freedom?$[8 + 6]$

4. Let Y be a response variable and $X_1, \ldots, X_k$ be covariates. Also, let ρ_i denote the correlation coefficient between Y and X_i, and let R denote the multiple correlation coefficient between Y and $X_1, \ldots, X_k$.
(a) Show that $R \ge \max\{|r_i|, 1 \le i \le k\}$.
(b) What is the exact relationship between R and r_i's when $k = 1$? [5+5]